Hierarchy problem and the cosmological constant in a five-dimensional Brans-Dicke brane world model

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Abstract

We discuss a new solution, admitting the existence of dS_4 branes, in five-dimensional Brans-Dicke theory. It is shown that, due to a special form of a bulk scalar field potential, for certain values of the model parameters the effective cosmological constant can be made small on the brane, where the hierarchy problem of gravitational interaction is solved. We also discuss new stabilization mechanism which is based on the use of auxiliary fields.

1 Introduction

Brane world models and their phenomenology have been widely discussed in the last years. The most known models – the Arkani-Hamed, Dimopoulos and Dvali scenario [1] and the Randall-Sundrum model [2], – provide elegant, although different, solutions to the hierarchy problem of gravitational interaction. It seems that the second model is more consistent, because it takes into account the proper gravitational field of the branes. Meanwhile it was shown that the four-dimensional effective theory on the branes in the Randall-Sundrum model contains a massless scalar field – the radion, which is a consequence of the fact that the distance between the branes is not fixed by the parameters of the model. The coupling constant of this field to matter on the negative tension brane, which is assumed to trap the Standard Model fields, appears to be very large, which contradicts experimental data even at the level of classical experiments [3, 4].

This problem was solved by introducing an extra scalar field living in the bulk. The most consistent model was proposed in paper [5], where exact solutions to equations of motion for the background metric and the scalar field were found. The size of the extra dimension is defined by the boundary conditions for the scalar field on the branes.

The models discussed above assume the metric on the branes to be the flat Minkowski metric. At the same time it is evident that more realistic models should account for a cosmological evolution on the branes. This problem is widely discussed in scientific literature, see, for example, papers [5, 6], reviews [7, 8] and references therein. Quite an interesting class of the brane world models is the one describing background solutions with dS_4 metric on the branes. There are examples of such solutions in a slightly modified Randall-Sundrum model (with non-equal brane tensions) [9, 10], as well as in models with additional matter on the branes and in the bulk [11] including scalar field living in the bulk [12, 13]. The latter models are of particular interest, because an additional scalar field can fix the size of the extra dimension thus giving stabilized models.

One of the standard scalar-tensor theories of gravity is the Brans-Dicke theory (see [14, 15]). In the context of brane world models this theory was also discussed in the literature for the case of static [16] and time dependent [17, 18] solutions, including background solutions with dS_4 metric on the branes. Although one can transform the theory to the Einstein frame, in which the scalar field minimally couples to gravity, the theory in the original form can provide elegant

and simple background solutions. Moreover, in principle it is possible that we live in the world, in which five-dimensional scalar field is non-minimally coupled to five-dimensional curvature (this is defined by the interaction of matter on the branes with gravity, i.e. by the metric which we are supposed to perceive). In this case it is more convenient to consider untransformed action in the Jordan frame, in which the scalar field non-minimally couples to gravity.

As it was mentioned above, brane world models are of particular interest because they provide elegant solution to the hierarchy problem of gravitational interaction. Nevertheless, such models should also describe cosmological evolution at least on the late stages. In this paper we discuss a stabilized brane world model in five-dimensional Brans-Dicke theory admitting dS_4 branes. We also discuss the values of fundamental parameters, which can make the effective cosmological constant on the brane which is supposed to contain SM fields (and where the hierarchy problem of gravitational interaction is solved) very small.

2 The model

Let us consider gravity in a five-dimensional space-time $E = M_4 \times S^1/Z_2$, interacting with two branes and with the scalar field ϕ. Let us denote coordinates in E by $\{x^M\} = \{t, x^i, y\}$, $M = 0,1,2,3,4$, where $x^0 \equiv t$; $\{x^i\}$, $i = 1,2,3$ are three-dimensional spatial coordinates and the coordinate $y \equiv x^4$, $-L \le y \le L$, corresponds to the extra dimension. The extra dimension forms the orbifold S^1/Z_2, which is a circle of diameter $2L/\pi$ with the points y and $-y$ identified. Correspondingly, the metric g_{MN} and the scalar field ϕ satisfy the orbifold symmetry conditions

$$\begin{aligned} g_{\mu\nu}(x,-y) = g_{\mu\nu}(x,y), \quad g_{\mu 4}(x,-y) = -g_{\mu 4}(x,y), \\ g_{44}(x,-y) = g_{44}(x,y), \quad \phi(x,-y) = \phi(x,y), \end{aligned} \tag{1}$$

$\mu = 0,1,2,3$. The branes are located at the fixed points of the orbifold $y = 0$ $y = L$.

The action of the model has the form

$$\begin{aligned} S = \int d^4x \int_{-L}^{L} dy \sqrt{-g}\left[\phi R - \frac{\omega}{\phi} g^{MN}\partial_M\phi\partial_N\phi - V(\phi)\right] - \\ - \int_{y=0} \sqrt{-\tilde{g}}\lambda_1(\phi) d^4x - \int_{y=L} \sqrt{-\tilde{g}}\lambda_2(\phi) d^4x. \end{aligned} \tag{2}$$

Here $V(\phi)$ is the scalar field potential in five-dimensional space-time, $\lambda_{1,2}(\phi)$ are scalar field potentials on the branes, ω is the five-dimensional Brans-Dicke parameter (we suppose that $\omega \gg 1$), $\tilde{g}_{\mu\nu}$ denotes induced metric on the branes. The signature of the metric g_{MN} is chosen to be $(-,+,+,+,+)$. Subscripts 1 and 2 label the branes. We also note that the dimension of the field ϕ is $[mass]^3$.

We consider the following standard form of the background metric, which is often used in brane world models (see, for example, [5])

$$ds^2 = \gamma_{MN} dx^M dx^N = e^{-2A(y)}\left(-dt^2 + a^2(t)\eta_{ij}dx^i dx^j\right) + dy^2 \tag{3}$$

with $\eta_{ij} = diag(1,1,1)$, and the following form of the background solution for the scalar field

$$\phi(x,y) = \phi(y) \tag{4}$$

(the background solution is the solution corresponding to the vacuum states on the branes, i.e. when all the fields on the branes are in their vacuum states).

In paper [16] a model, admitting a simple background solution in the case $a(t) \equiv 1$, was proposed. Indeed, with

$$V(\phi) = \Lambda\phi \tag{5}$$

the background solution for the metric and scalar field takes a simple form [16]:

$$A(y) = k|y| - kL,$$

$$\phi = v\left(e^{-A}\right)^{\frac{1}{\omega+1}}, \tag{6}$$

where L is the size of the extra dimension, which is defined by boundary conditions on the branes together with constant v, and k is given by

$$k^2 = -\Lambda\frac{(\omega+1)^2}{(3\omega+4)(4\omega+5)}. \tag{7}$$

Parameter Λ (and consequently parameter k) characterizes the energy scale of five-dimensional gravity.

It appears that it is possible to modify slightly the bulk potential (5) to get dS_4 background metric on the branes. We consider ansatz (3), (4) for the vacuum background solution with

$$a(t) = e^{Ht}, \tag{8}$$

$$\phi = v\left(e^{-A}\right)^{\frac{1}{\omega+1}}, \tag{9}$$

where H is the four-dimensional Hubble parameter on the branes, v is a constant. The Hubble parameter H and the constant v will acquire their values after solving the corresponding equations of motion. Note that the form of the background solution for the scalar field is the same as the one used in [16] for the case $a(t) \equiv 1$ (see equation (6)). Assumption (9) simplifies considerably equations of motion for the scalar field and the Einstein equations following from (2).

The Einstein equations for a general form of the metric and the equation of motion for the Brans-Dicke scalar field can be found, for example, in [17] (these equations are obtained for $\phi = \varphi^2/(8\pi)$ and $\lambda_{1,2}(\phi) = 0$, nevertheless, the case $\lambda_{1,2}(\phi) \neq 0$ can be easily restored). These equations have quite a complicated form. But for our choice of the background solution (3), (8) and (9) the corresponding equations take a simpler form:

1. $\mu\nu$-component

$$6H^2e^{2A} + \frac{6\omega+8}{\omega+1}A'' - A'^2\frac{12\omega^2+31\omega+20}{(\omega+1)^2} = \\ = \frac{V(\phi)}{\phi} + \frac{\lambda_1(\phi)}{\phi}\delta(y) + \frac{\lambda_2(\phi)}{\phi}\delta(y-L), \tag{10}$$

2. 44-component

$$12H^2e^{2A} - A'^2\frac{12\omega^2+31\omega+20}{(\omega+1)^2} = \frac{V(\phi)}{\phi}, \tag{11}$$

3. equation for the field ϕ

$$12H^2e^{2A}+\frac{6\omega+8}{\omega+1}A''-A'^2\frac{12\omega^2+31\omega+20}{(\omega+1)^2}= \tag{12}$$
$$=\frac{dV(\phi)}{d\phi}+\frac{d\lambda_1(\phi)}{d\phi}\delta(y)+\frac{d\lambda_2(\phi)}{d\phi}\delta(y-L),$$

where $'=\frac{d}{dy}$.

We suppose that the bulk scalar field potential has the form

$$V(\phi)=\Lambda\phi+\frac{\beta}{\phi^{2\omega+1}}, \tag{13}$$

where $\Lambda<0$, $\beta<0$. The physical motivation of such a choice of the potential is the following: the first term is responsible for the solution to the hierarchy problem of gravitational interaction on the brane at $y=L$, whereas the second term is responsible for the constant Hubble parameter on the branes (i.e. dS_4 space-time on the branes). One can see that the second term in (13) is fine-tuned because the five-dimensional Brans-Dicke parameter ω is utilized in the definition of potential (13). This fine-tuning of the five-dimensional bulk scalar field potential is similar to the choice of cosmological constant in four-dimensional gravity to get dS_4 space-time. Note that the vacuum structure of the four-dimensional space-time on the branes is defined by the bulk scalar field potential, not by the potentials on the branes. We also suppose that parameters β, Λ are in the TeV energy range and do not contain hierarchical difference.

First, let us consider equations (10), (11) and (12) in the bulk. The corresponding solution to these equations of motion has the form

$$A=-\ln\left(C_1e^{k|y|}+C_2e^{-k|y|}\right) \tag{14}$$

with k defined by equation (7). We also can get

$$H^2=-4k^2\frac{3\omega+4}{3\omega+3}C_1C_2 \tag{15}$$

and

$$H^2=-\beta\frac{(\omega+1)}{3v^{2\omega+2}}, \tag{16}$$

which means that the four-dimensional Hubble parameter is expressed through the parameter β of the bulk scalar field potential, the five-dimensional Brans-Dicke parameter ω and the constant v, which will be defined below.

Let us discuss the values of the constants C_1, C_2. We are interested in the effective theory on the brane at $y=L$ (it will be shown below that the hierarchy problem is solved on this brane). To this end we should take such C_1, C_2 that make the four-dimensional coordinates on this brane Galilean (see [3, 4, 19]), i.e. $A|_{y=L}=0$. Thus,

$$C_1e^{kL}+C_2e^{-kL}=1. \tag{17}$$

Equations (15) and (17) define the values of C_1, C_2 (we suppose, that the Hubble parameter $H\ll k$ because k is in the TeV range, whereas H should correspond to the late time accelerated expansion).

$$C_2=e^{kL}\frac{1+\sqrt{1+\frac{(3\omega+3)H^2}{(3\omega+4)k^2}}}{2}\approx e^{kL}\left(1+\frac{3(\omega+1)H^2}{4(3\omega+4)k^2}\right), \tag{18}$$

$$C_1 \approx -e^{-kL}\frac{3(\omega+1)H^2}{4(3\omega+4)k^2}, \tag{19}$$

which results in

$$\begin{aligned} e^{-A} &\approx e^{(kL-k|y|)} + \frac{3(\omega+1)H^2}{2(3\omega+4)k^2}\sinh\left(kL-k|y|\right) = \\ &= e^{(kL-k|y|)} + \frac{(4\omega+5)\beta}{2\Lambda v^{2\omega+2}}\sinh\left(kL-k|y|\right). \end{aligned} \tag{20}$$

Now let us turn to the boundary conditions on the branes. They can be easily obtained from (10), (12) by the standard procedure (see, for example, [5]) and have the form

$$\begin{aligned} 2\frac{6\omega+8}{\omega+1}A'|_{y=+0} &= \frac{\lambda_1(\phi)}{\phi}\,|_{\phi=\phi(0)}\ , \\ 2\frac{6\omega+8}{\omega+1}A'|_{y=L-0} &= -\frac{\lambda_2(\phi)}{\phi}|_{\phi=\phi(L)}, \end{aligned} \tag{21}$$

$$\begin{aligned} 2\frac{6\omega+8}{\omega+1}A'|_{y=+0} &= \frac{d\lambda_1(\phi)}{d\phi}|_{\phi=\phi(0)}, \\ 2\frac{6\omega+8}{\omega+1}A'|_{y=L-0} &= -\frac{d\lambda_2(\phi)}{d\phi}|_{\phi=\phi(L)}. \end{aligned} \tag{22}$$

Note that there are no boundary conditions coming from equation (11) because it does not contain terms $\sim A''$ and $\sim \delta(y)$.

We will consider the following form of the potential on the branes

$$\lambda_{1,2}(\phi) = \pm 4\sqrt{3\omega+4}\sqrt{-\frac{\Lambda\phi^2}{4\omega+5} - \frac{\beta}{\phi^{2\omega}}} + F_{1,2}(x)\left(\phi-\phi_{1,2}\right), \tag{23}$$

where $F_{1,2}(x)$ are scalar fields and $\phi_{1,2}$ are constants. The absence of the kinetic terms for the fields $F_{1,2}(x)$ looks rather strange. Nevertheless one can recall that supersymmetry is based on the use of such "auxiliary" fields, which are necessary for reaching the closure of the supersymmetry algebra [20]. A simple example with the fields of such type in classical field theory can be also found in [20].

Of course, one can use a more conventional form of the stabilizing potentials, for example

$$\begin{aligned} \lambda_1(\phi) &= 4\sqrt{3\omega+4}\sqrt{-\frac{\Lambda\phi^2}{4\omega+5} - \frac{\beta}{\phi^{2\omega}}} - \frac{4(\omega+1)\sqrt{3\omega+4}\beta}{\sqrt{-\frac{\Lambda\phi_1^{4\omega+4}}{4\omega+5} - \beta\phi_1^{2\omega+2}}}\left(\phi-\phi_1\right) + \\ &+ q_1^2\left(\phi-\phi_1\right)^2, \end{aligned} \tag{24}$$

$$\begin{aligned} \lambda_2(\phi) &= -4\sqrt{3\omega+4}\sqrt{-\frac{\Lambda\phi^2}{4\omega+5} - \frac{\beta}{\phi^{2\omega}}} + \frac{4(\omega+1)\sqrt{3\omega+4}\beta}{\sqrt{-\frac{\Lambda\phi_2^{4\omega+4}}{4\omega+5} - \beta\phi_2^{2\omega+2}}}\left(\phi-\phi_2\right) + \\ &+ q_2^2\left(\phi-\phi_2\right)^2. \end{aligned} \tag{25}$$

The terms $\sim q_{1,2}^2$ usually are introduced to ensure the absence of tachyonic modes in the linearized theory [21]. But such form of the potentials appears to be highly fine-tuned. One

can see that there is strong relation between the terms even if we consider only one of the brane potentials ((24) or (25)). The auxiliary fields add extra degrees of freedom, which make the inner fine-tuning of the terms in the brane potentials unnecessary. Nevertheless, there remains fine-tuning in (23) – the first term in (23) is defined by the form of the bulk scalar field potential and by the form of the background metric (3) (one can check it by straightforward calculations). Such fine-tuning is inherent to almost all brane world models with compact extra dimension and two branes (see, for example, [2, 5]).

Equations of motions for fields $F_{1,2}(x)$ give (which can be obtained by means of the standard variation procedure with respect to the fields $F_{1,2}(x)$)

$$\phi|_{y=0} = \phi_1, \tag{26}$$
$$\phi|_{y=L} = \phi_2. \tag{27}$$

We see that these equations do not contain fields $F_{1,2}(x)$ itself. Equations (21) appear to be satisfied automatically for the choice (23) (one can check it using (11), (13) and (16)), whereas equations (22) define the background values of the fields $F_{1,2}(x)$

$$F_1 = -\frac{4(\omega+1)\sqrt{3\omega+4}\beta}{\sqrt{-\frac{\Lambda\phi_1^{4\omega+4}}{4\omega+5} - \beta\phi_1^{2\omega+2}}}, \tag{28}$$
$$F_2 = \frac{4(\omega+1)\sqrt{3\omega+4}\beta}{\sqrt{-\frac{\Lambda\phi_2^{4\omega+4}}{4\omega+5} - \beta\phi_2^{2\omega+2}}} \tag{29}$$

(we see, that the fields $F_{1,2}(x)$ appear in equation of motion for the scalar field ϕ). Thus, although fields $F_{1,2}(x)$ do not contain kinetic terms and are not dynamical, they can be treated as normal fields and one can use the standard variation technique to obtain corresponding equations of motion [20].

Equations (26) and (27) define the size of the extra dimension. Indeed, from (9), (20) and (27) it follows that $v = \phi_2$. Then, neglecting the contribution of the term proportional to H^2/k^2 (because we suppose that $H \ll k$), we get from (9) and (26) (see [16] for details)

$$L \approx \frac{(\omega+1)}{k} \ln\left(\frac{\phi_1}{\phi_2}\right). \tag{30}$$

Note that the same relation was obtained for the simpler model discussed in [16]. The four-dimensional Planck mass on the brane at $y = L$ can also be easily obtained. For our purposes the contribution proportional to H^2/k^2 in (9), (20) can be also neglected, and in this approximation the corresponding Planck mass is (detailed derivation can be found in [16])

$$M_{Pl}^2 = \frac{1}{2}\int_{-L}^{L} \phi e^{-2A} dy \approx \frac{\phi_1}{2k} e^{2kL}. \tag{31}$$

Thus, if $kL \approx 35$ and parameters ϕ_1, k of the theory lie in the TeV range, the hierarchy problem on the brane at $y = L$ is solved in the way analogous to that used in the original Randall-Sundrum model [2]. Thus, we get weak four-dimensional gravity on the brane at $y = L$ with $M_{Pl} \sim 10^{19}\, GeV$, whereas five-dimensional gravity is characterized by the TeV energy scale.

The five-dimensional Hubble parameter on the brane is defined by equation (16), which can be rewritten as

$$H^2 = -\beta \frac{(\omega+1)}{3\phi_2^{2\omega+2}}. \tag{32}$$

If we suppose that $\beta \approx -1\,TeV^{6\omega+8}$, $\omega \approx 45$ and $\phi_2 \approx 10\,TeV^3$, then the Hubble parameter in (32) has the same small value as that in ordinary four-dimensional gravity defined by vacuum energy density

$$\rho_\Lambda \sim H^2 M_{Pl}^2 \sim 10^{-47} GeV^4. \tag{33}$$

Such value of H can correspond to the late time accelerated expansion of the Universe if we suppose that our four-dimensional world lives on the brane at $y = L$.

Note, that such a small value of the effective cosmological constant appears because of the large power $\sim 2\omega$ in the denominator of the fine-tuned bulk potential (13). One can argue that such a way of obtaining a small effective cosmological constant is analogous to introducing its small value "by hand". It is not the case in the model discussed above. It is somewhat similar to the solution of the hierarchy problem of gravitational interaction in the Arkani-Hamed, Dimopoulos and Dvali scenario [1], where four-dimensional Planck mass on the brane appears to be large due to a large number of extra dimensions. But it is necessary to note that the large value $10^{2\omega+2}$ is not introduced to the model by hand. Indeed, if $\phi_2 \approx 1\,TeV^3$, then from (32) it follows that $H^2 \approx 15\,TeV^2$, which is extremely large value in comparison with the present day value of the Hubble parameter. Thus, the effective four-dimensional Hubble parameter on the brane depends on the value of the scalar field ϕ on the brane. The value $\phi_2 = 10\,TeV^3$ does not create a new hierarchy itself.

3 Stability

A consistent study of stability of our model, at least in the linear approximation, appears to be quite a complicated task, which goes beyond the scope of this paper. Indeed, one should derive linearized equations of motion above the background solution, isolate the physical degrees of freedom and find the mass spectra of the excitations. Nevertheless we can simplify the problem and consider the simpler case $\beta = 0$, for which we can show that our method of fixing the size of the extra dimension by utilizing the auxiliary fields $F_{1,2}(x)$ does not lead to instabilities at least in the simplest cases. For $\beta = 0$ the potentials on the branes and the bulk potential take the form (compare with those used in [16]):

$$V(\phi) = \Lambda\phi, \qquad \lambda_{1,2}(\phi) = \pm 4\sqrt{-\Lambda}\sqrt{\frac{3\omega+4}{4\omega+5}}\phi + F_{1,2}(x)\left(\phi - \phi_{1,2}\right). \tag{34}$$

Boundary conditions (21) and (22) lead to the following background values

$$F_1 \equiv 0, \tag{35}$$
$$F_2 \equiv 0, \tag{36}$$

which also follow from (28), (29) for $\beta = 0$.

Linearized gravity in five-dimensional Brans-Dicke stabilized brane world models was thoroughly examined in [21]. In particular, the model with

$$V(\phi) = \Lambda\phi, \qquad \lambda_{1,2} = \pm 4\sqrt{-\Lambda}\sqrt{\tfrac{3\omega+4}{4\omega+5}}\phi + \tfrac{\beta_{1,2}^2}{2}(\phi - v_{1,2})^2 \tag{37}$$

was also considered. It was shown that this model is stable and does not contain the scalar zero mode. The only difference between (34) and (37) is the stabilizing potentials on the branes (such potentials are often called stabilizing potentials because they define the values of the scalar field on the branes, which leads to fixation of the size of the extra dimension and thus to stabilized model). The results obtained in [21] show that from the four-dimensional point of view linearized gravity can be described by the tensor (massless and massive tensor gravitons) and scalar (massive scalar modes) physical degrees of freedom. The tensor sector does not depend on the form of the stabilizing potentials and, as it was shown in [21], does not contain tachyons. But the scalar sector of the model changes under the change of the stabilizing potentials. We will use results of [21] to show that the new method of stabilization does not lead to any new unwanted consequences in the scalar sector. We will not present here the full set of linearized equations of motion because these equations are quite tedious. One can find detailed calculations in [21].

To start with, let us parameterize the metric and the scalar field as

$$g_{MN}(x,y) = \gamma_{MN}(y) + h_{MN}(x,y), \tag{38}$$

$$\phi(x,y) = \phi_0(y) + f(x,y), \tag{39}$$

$$F_{1,2}(x) = F^0_{1,2} + j_{1,2}(x) = j_{1,2}(x). \tag{40}$$

For consistency with the previous sections below we will write $F_{1,2}$ and $\phi(y)$ for the background solution instead of $F^0_{1,2}$ and $\phi_0(y)$.

It was shown in [21] that the physical degrees of freedom of the scalar sector for any form of the bulk scalar field potential $V(\phi)$ can be completely described by the new field g:

$$g(x,y) = e^{-2A}\phi^{2/3}\left(h_{44}(x,y) + \frac{2}{3}\frac{f(x,y)}{\phi}\right).$$

It was also shown that the fluctuations f of the stabilizing scalar field ϕ can be expressed in terms of the field g through the gauge condition [21]

$$g' = \frac{4}{3}\left(\omega + \frac{4}{3}\right)e^{-2A}\frac{\phi'}{\phi^{4/3}}f. \tag{41}$$

The corresponding equation of motion for the field g in the bulk looks like [21]

$$\left(g'\frac{\phi^{5/3}e^{2A}}{\phi'^2}\right)' - \frac{2}{9}\left(3\omega+4\right)\frac{e^{2A}}{\phi^{1/3}}g + \frac{\phi^{5/3}e^{2A}}{\phi'^2}\partial_\mu\partial^\mu g = 0. \tag{42}$$

The difference between the model examined in [21] and the model under consideration is the brane scalar field potentials. Indeed, from equations (26), (27) it follows that $f|_{y=0} = f|_{y=L} = 0$ and thus using (41) we get boundary conditions

$$\begin{aligned} g'|_{y=+0} &= 0, \\ g'|_{y=L-0} &= 0. \end{aligned} \tag{43}$$

The latter conditions differ from those obtained in [21] for (37).

There are also "boundary conditions", following from the linearized equation of motion for the field f [21], which, for our choice of the brane scalar field potentials (34), take the form

$$\begin{aligned} j_1(x) &= -\tfrac{3\phi^{1/3}}{\phi'}e^{2A}\partial_\mu\partial^\mu g|_{y=+0}, \\ j_2(x) &= \tfrac{3\phi^{1/3}}{\phi'}e^{2A}\partial_\mu\partial^\mu g|_{y=L-0}. \end{aligned} \tag{44}$$

We see from (44) that the fields $j_1(x)$, $j_2(x)$ are not dynamical and appear to be defined by the values of the field g on the branes. The system appears to be not overconstrained.

To make the mode decomposition, we represent $g(x,y)$ in a standard way [21]:

$$g(x,y) = \sum_{n=1}^{\infty} \varphi_n(x) g_n(y), \qquad \eta^{\mu\nu}\partial_\mu\partial_\nu\varphi_n(x) = \mu_n^2\varphi_n(x).$$

Now the spectrum of scalar modes is defined by equations

$$\left(g_n' \frac{\phi^{5/3}e^{2A}}{\phi'^2}\right)' - \frac{2}{9}\left(3\omega+4\right)\frac{e^{2A}}{\phi^{1/3}} g_n + \frac{\phi^{5/3}e^{4A}}{\phi'^2}\mu_n^2 g_n = 0, \tag{45}$$

$$\begin{aligned} g_n'|_{y=+0} &= 0, \\ g_n'|_{y=L-0} &= 0, \end{aligned} \tag{46}$$

where $g_n(y)$ is the wave function of the four-dimensional mode with the mass μ_n [21]. It is not difficult to show (multiplying (42) by g_n and integrating it in the limits $0 < y < L$) that $\mu_n^2 > 0$. As for the zero mode with $\mu_0 = 0$, its wave function $g_0 \equiv 0$. Thus, there is no zero scalar mode, which is inherent to stabilized brane world models.

We showed that the model with $\beta = 0$ is stable at least under the small fluctuations of the fields and our method of fixing the size of the extra dimension does not lead to instabilities. Other properties of the spectrum, such as orthogonality of the wave functions g_n, can be easily obtained using the results of [21]. As for the case $\beta \neq 0$, for a small H corresponding to (33) one also expects stability of the model. Indeed, if all parameters of the model with $\beta = 0$ lie in the TeV range, the masses of the lowest modes in the four-dimensional effective theory on the brane are also expected to lie in the TeV range, as well as the mass gaps between the modes. The case $\beta \neq 0$, providing the present day value of H, in principle leads to the modification of the spectra of tensor and scalar modes, but this modification can be neglected because of the extremely small value of H in comparison with TeV energy scale. In other words, we expect practically the same tower of the modes (including the massless graviton) propagating in the dS_4 space-time instead of the flat Minkowsky space-time, and it does not pose any problems with stability (we could expect ghost modes if the masses of the tensor modes were smaller than $2H^2$, as it happens in four-dimensional massive gravity [22], but it is not the case under consideration). Locally we can even neglect the influence of the non-zero Hubble parameter (for example, when considering the collider phenomenology or Newtonian gravity). For these reasons the model proposed in this paper seems to be stable.

4 Conclusion

In this paper we discussed stabilized brane world model in five-dimensional Brans-Dicke theory. The choice of the bulk potential was motivated by the demand to have dS_4 space-time on the branes in the vacuum state (i.e. when there are no any matter fields on the branes). We note, that our solution is stationary, because the background solution for the scalar field does not depend on time, the size of the extra dimension is fixed and the four-dimensional Hubble parameter is constant. Such situation is realized in the limit $x^0 \to \infty$. Indeed, in this case the ordinary and dark matter average densities on the branes tend to zero, solutions for the metric and the scalar field tend to this background solution.

Nevertheless the appropriate form of the bulk and brane scalar field potential should be fine-tuned in order to get dS_4 space-time on the branes. As it was mentioned above, such fine-tuning is the price we have to pay in order to get solutions with desired properties. The fine-tuning of the bulk potential is in some sense analogous to the choice of cosmological constant in ordinary four-dimensional gravity in order to get maximally symmetric dS_4 space-time, whereas the fine-tuning of the brane potentials is necessary for the self-consistency of the solution.

The advantages of the model presented above are the following.

1. The hierarchy of gravitational interaction is solved in the model in the way analogous to the one utilized in the original Randall-Sundrum model [2] (see equation (31)).

2. The size of the extra dimension is fixed (see equation (30)).

3. The small four-dimensional Hubble parameter is defined by the parameter ϕ_2 and can account for the late time accelerated expansion on the brane.

We note that the effective cosmological constant on the branes appears to be small because of the large power in the denominator of the second term in the bulk scalar field potential (13), which is similar in some sense to the solution of the hierarchy problem of gravitational interaction in the Arkani-Hamed, Dimopoulos and Dvali scenario [1]. As for the fine-tuning of the scalar field potentials, in a most stabilized five-dimensional brane world models one should use special forms of the potential to get stationary vacuum solution, and the five-dimensional Brans-Dicke theory is not an exception.

We hope that the results presented in this paper can be interesting for a future investigations of brane world models.

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